# Best Practices in Mathematics Instruction: Teaching for Understanding

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*At some point between elementary and junior high school, something seems to occur that generates a rooted dislike toward the scientific disciplines in general and mathematics in particular. This results in most students in developmental education associating the word math with a series of illogical rules, totally disconnected from reality and common sense. As teachers and developmental educators, we need to take responsibility for the situation and create a teaching methodology that emphasizes the understanding of mathematical concepts rather than their rote and disconnected application. Extending results obtained for the general population of underprepared students, this article outlines practical techniques that can be used to make understanding the focal point of mathematical instruction.*

Isn't it frustrating when the majority of the students in the class considers mathematics as a collection of disconnected rules, which make absolutely no sense? Such an attitude reflects their strong dislike for the subject, which probably originates from previous negative experiences and from the failure to grasp the ideas and to use the connections between them. When understanding is not present, interest fades and memorization becomes a temporary mean to produce answers, setting the stage for post-secondary students who lack motivation and background preparation. Research on underprepared students who struggle at the college level shows that they have the tendency to study just prior to exams or due dates, memorize terms but cannot recognize them in related examples, and lack the ability to see how course components interrelate. Furthermore, they are incapable of monitoring their understanding of the various concepts, cannot articulate what they have studied and do not try to connect and elaborate knowledge (Klopfleisch, 2005; Weinstein, 1982).

When the students' involvement remains superficial and detached, simply confined to listening to a lecture and reading material from the textbook, learning does not occur. Research studies have established that

learning does not happen overnight, but occurs over an extended period of time as a result of writing and thinking about what one is learning, relating it to past experiences, and applying it in real-life situations (Felder & Brent, 1996). Therefore, learners cannot be passive spectators that just listen to a lecture. To be successful, they must be actively engaged in the learning process, so that they can incorporate what they learn and make it a part of themselves. When strategic thinking processes are used, students obtain positive outcomes such as higher grades and increased retention rates (Weinstein, Dierking, Husman, Roska, & Powdrill, 1998).

Knowing how to solve first-degree equations is different from understanding equations, their usefulness and versatility, and the principles that are used in solving them. Understanding a concept translates into being able to perform a variety of thought-demanding tasks, such as explain, find examples, generalize, and apply it to practical situations (Perkins, 1993). Of course, knowledge is the basic prerequisite to understanding, but the gap between knowledge and understanding cannot be filled unless teaching is aimed toward the upper levels of Bloom's taxonomy. When the goal of teaching expands from mere knowledge toward comprehension, application, analysis, and synthesis, students become immersed in a culture of thinking and move beyond simple recollection of facts.

## Suggestions for Useful Practices

To teach for understanding we must broaden our repertoire of activities beyond conventional knowledge sphere drills and calculation exercises to include thought-demanding problems and applications. By asking more challenging questions, we will stimulate comprehension of concepts, as well as analytical and thinking skills, helping to develop citizens that are better prepared for their future professions in our fast changing world. The following techniques focus on making understanding the true essence of mathematical teaching.

### *Compare and contrast a new concept with one or more previously learned concepts.*

Students tend to be less intimidated by a new idea when they can see how it connects with material that is already familiar. Such connection is critical in facilitating assimilation of new concepts, by creating a meaningful context where new ideas fit harmoniously in an expanded frame. Emphasizing how a new concept fits with previously learned ideas and the advantages of the newly gained perspective, opens the road to new insights and encourages students to formulate new hypotheses and make

valid inferences. Comparing new and old ideas requires students to recall previous knowledge and apply it to a different situation, engaging the learners in critical thinking and deepening their understanding.

***Create a conducive learning environment by using a mix of instructional methods.***

To establish a positive environment from the beginning of a course, students must know the instructor's goals, expectations, and evaluation criteria (Casazza & Silverman, 1996). Clearly stated goals and expectations set the tone for the right classroom atmosphere. Students' motivation and interest are boosted when the instructor is capable of creating a conducive learning environment. Such environment is made up of many factors, including enthusiasm, confidence in the students' abilities, and instructional methods. Evidence collected in a number of studies conducted among college students points out that the dominant learning style is visual or iconic, followed by hands-on or learning-by-doing (Lemire, 1998). In addition, the use of a variety of instructional methods increases the chances of success of underprepared college students (Casazza & Silverman, 1996). Using a variety of instructional methods not only emphasizes understanding instead of memorization, but is also more likely to appeal to the learning styles of our learners.

***Challenge students with questions beyond the recollection level.***

Assigning some problems aimed beyond simple recollection skills stimulates critical thinking among the students and fosters their analytical abilities. Thought-provoking questions make the subject more interesting and stimulate curiosity and participation. Such questions communicate one's belief in the abilities of the learners, implying that the teacher has high expectations of the students. Learners generally tend to rise to the level required of them, so teachers should not be afraid to challenge their students with a variety of questions that extend toward analysis and application. Providing opportunities for review and class discussion through extra tutorial sessions and organizing study groups led by peer students are two of the techniques that are generally successful in preparing the class for the task.

***Encourage students to write about mathematics.***

Post-secondary students usually approach mathematics with a high level of emotion. Many will confess that they simply "don't understand it" and are "not capable" of achieving satisfactory results. One way to dispel their rooted fear is to ask them to write and reflect about their attitude

toward the subject. Journal writing exercises encourage self-reflection and stimulate a proactive approach. Students who have a negative self-perception of their mathematical abilities tend to discharge their emotions when writing about them. Their energies can then be channeled toward the development of positive strategies for success in mathematics. Such journals can contain an introductory page that emphasizes basic study skill implementation strategies and the students can be asked to reflect on ways they can use such strategies in the course they are presently taking.

***Provide continuous assessment and prompt feedback.***

A structured environment is generally conducive to learning and particularly beneficial to at-risk students (Boylan, Bonham, Claxton, & Bliss, 1992). Assessment and feedback are central components of learning and must be offered regularly, so that students can benefit from them. Frequent assessment in the form of quizzes, assignments, and tests eliminates procrastination and forces the students to make an effort to keep up with the material. To accomplish this task, it is crucial to ensure compatibility between classroom and laboratory activities, so that both course components complement each other toward the achievement of the learning objectives (Boylan & Saxon, 1998). Laboratory activities that are connected to the class material and provide opportunities for reflection, foster understanding of concepts, and assimilation of the material.

Once assessment has been completed, the most important aspect of feedback consists of pointing out common mistakes and helping students discover the overgeneralizations that cause those errors. Teachers must analyze the mistakes of their students and search for possible causes so that they can modify the presentation of the related topics, emphasizing their correct understanding and individuating the idea that is missed, in an attempt to eliminate misinterpretations of concepts.

***Support learning with conceptual models.***

Diagrams, flow charts, mental pictures, similarities-differences tables, and any form of supporting model that represents the information symbolically, are powerful tools that promote deeper understanding of the concepts and trigger the ability to make connections. Conceptual models compel students to rearrange and elaborate the material in their own way, so that ideas are seen from a different perspective and connections emerge. Models engage the students' thinking skills and support

interpretation and assimilation of the concepts. Encouraging learners to build their own diagrams and conceptual models, reinforces their understanding of the material and boosts their recollection capabilities.

***Teach the discipline as a whole entity.***

Concepts and principles in a discipline are not understood in isolation (Perkins, 1993). We should aim at transmitting a mathematical system of thought, in which principles and concepts flow and function harmoniously. Constructing an integral view of the discipline and its specific methods assists the students in the development of a scientific mentality and helps them apply concepts and principles across the sciences. Such an approach is crucial in fostering the growth of a mathematical thinking process, and augments the ability to think independently and to solve problems.

I like to compare knowledge to a puzzle and concepts to the single pieces of the puzzle. Looking too closely at the details generates the risk of losing sight of the whole picture, and without it the details are insignificant. The whole structure of the discipline must be the constant focus, while the single concepts must integrate and connect meaningfully with it, becoming indistinguishable from the conceptual frame of the discipline.

***Teach for transfer.***

Research findings agree on the fact that students fail to carry over to different contexts, ideas that were presented to them in a specific context (Perkins, 1993). To combat this tendency, we must first of all be well aware of it, and, subsequently, we must train the students to make more connections with other disciplines and situations. We can, for instance, provide examples in different settings, so that the learners can transcend the artificial boundary of the topic and the circumstances of initial acquisition and expand the ideas beyond. When teaching equation solving, we can, for example, point out that inverting formulas follows exactly the same principle. We can then provide examples and applications that span geometry, chemistry, and physics, to name just a few fields of application. By doing this, we teach our learners to transfer a mathematical process to other subjects and situations, and, at the same time, we demonstrate a practical use of mathematics across other disciplines.

## Application of Best Practice Strategies

The *sign rule* for multiplying integers will serve as an application example for the previous strategies (see Figure 1).

$$\begin{array}{l} + \cdot + = + \\ + \cdot - = - \\ - \cdot + = - \\ - \cdot - = + \end{array}$$

*Figure 1*

***Compare and contrast the sign rule for multiplying integers with addition of even and odd numbers.*** Associate even with positive, and odd with negative. Notice that adding two even numbers or two odd numbers produces an even number, in the same way as multiplying two positives or two negatives gives a positive number. Similarly, adding an even with an odd number generates an odd number, in the same way as multiplying a positive with a negative produces a negative (see Figure 2).

| | | |
|---|---|---|
| Even + even = even | → | $+ \cdot + = +$ |
| Even + odd = odd | → | $+ \cdot - = -$ |
| Odd + even = odd | → | $- \cdot + = -$ |
| Odd + odd = even | → | $- \cdot - = +$ |

*Figure 2*

***Use a mix of instructional methods.*** A visual method approach often appeals to a majority of learners and provides a second justification of the sign rule. Build a rectangular two by three sheet of paper, leave one side white and the other dotted, and assign a sign to each face or color. Draw a system of coordinate axes and use flipping. Start placing the positive face in quadrant one. The area of the rectangle can be associated to the product of the integers $(+2)\cdot(+3) = +6$. Now flip the rectangle to quadrant two to obtain a negative (dotted) area, representing the product $(-2)\cdot(+3) = -6$. Flipping to quadrant three visualizes the positive product $(-2)\cdot(-3) = +6$, and finally quadrant four gives the visualization of $(+2)\cdot(-3) = -6$ (see Figure 3).

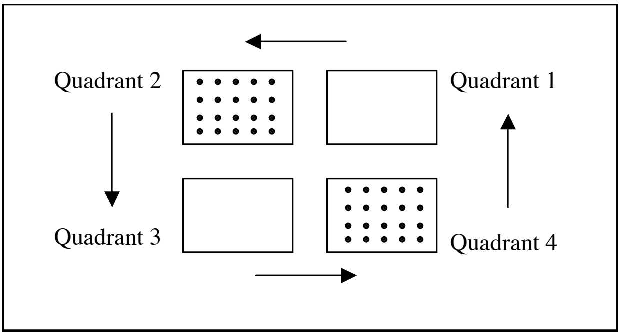


*Figure 3*

***Provide continuous assessment and prompt feedback.*** Have a series of assignments and quizzes on integer operations and problem solving. Make sure you include some terminology and questions at various skill levels.

***Support learning with conceptual models.*** Have students draw a flow chart of multiplication and division of integers, or alternatively create a summary table of their own design (see Figure 4).

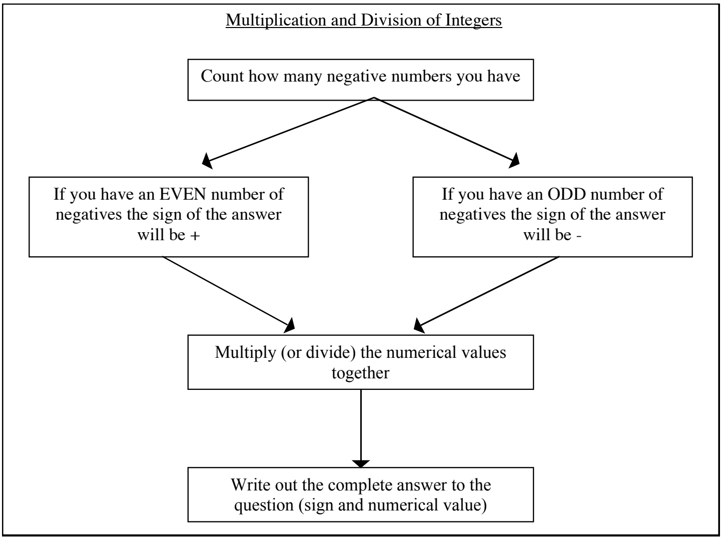


*Figure 4*

***Teach the discipline as a whole.*** Emphasize that integers are introduced because whole numbers are not closed under subtraction. We want to be able to perform as many operations as possible within each number set, so we introduce a new set that is closed under subtraction. Similarly, we will introduce fractions and rational numbers, because integers are not closed under division.

## Conclusion

The constant aim of teachers and developmental educators should be the development of critical thinking skills so that students are encouraged to shift from simple recollection of facts to understanding of concepts. When rote memorization is abandoned and ideas are understood, students have a chance to see the connections among the various concepts and to experience success instead of fear and frustration. Their negative attitude toward mathematics might finally leave room for more positive feelings.

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